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DESCRIPTION OF THE HORIZONTAL TEMPERATURE AND PRESSURE GRADIENT SCALE

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The horizontal temperature and pressure gradient scale has been designed to make possible accurate and rapid calculations of the various relationships between wind velocities and horizontal temperature and pressure gradients in the atmosphere. All the variables entering the equations are explicitly taken account of throughout their entire range of values, yet but one setting of the slide of the scale is required for any one calculation. For those relationships in which some of the variables, such as the isobar or isotherm interval used, the density (as given by the temperature and pressure) or the mean temperature, are constant or nearly constant for a large number of cases to be considered, the slide need be set but once for all the cases to which these constants apply. Convenient practical units are used throughout, with unit conversion scales included wherever feasible.

The balance of forces in a horizontal plane in the atmosphere with no externally impressed forces may be expressed in terms of the equations:

1. Geostrophic Wind: $fc = \frac{1}{\rho}\frac{\Delta p}{\Delta n}$

(The accleration due to the radius of curvature of the trajectories is neglected)

2. Cyclostropic Wind: $\frac{c^2}{R} = \frac{1}{\rho}\frac{\Delta p}{\Delta n}$

(The Coriolis acceleration is neglected)

3. Gradient Wind: $fc \pm \frac{c^2}{R} = \frac{1}{\rho}\frac{\Delta p}{\Delta n}$

(The + sign is to be taken with cyclonic curvature)
(The - sign is to be taken with anticyclonic curvature)

where

c = Wind velocity

f = Coriolis parameter = $2\Omega \sin \phi$.

ϕ = Latitude of the place considered.

Ω = angular velocity of the earth = 7.292×10^{-5} sec^{-1}.

ρ = density of the air considered, and $\frac{1}{\rho} = \frac{R}{m}\frac{T}{p}$.

R/m = Gas constant for dry air = 2.870×10^{6} ergs per gm per ^{o}C.

T = Temperature of the air considered.

p = Pressure of the air considered.

R = Radius of curvature (of the trajectory) of the air considered.

Δn = Spacing of the isobars, measured normal to the isobars.

Δp = Isobar interval for which the map considered is drawn.

Introducing the hydrostatic relationship the following approximate equations may be derived.

4. Thermal Gradient: $f\frac{dc}{dz} = \frac{g\Delta T}{T\Delta n}$

5. Advective Temperature Change: $f\frac{dc}{dz} = \frac{g}{C_n T}\frac{\partial T}{\partial t}$

6. Advective Pressure Change: $f\Delta c = \frac{1}{C_n \rho}\frac{\partial \Delta p}{\partial t}$

where

T = Mean temperature of the column considered.

ρ = Mean density of the column considered.

ΔT = Interval used for the mean isotherms of the column considered.

Δn = Horizontal spacing of the isotherms.

$\frac{\partial T}{\partial t}$ = Advective rate of change of the mean temperature of the column considered.

$\frac{\partial \Delta p}{\partial T}$ = Advective rate of change of the weight of the column considered.

$\frac{dc}{dz}$ = Shear of the wind velocity with height in the column considered.

Δc = Total change (determined vectorially) of the wind velocity from the bottom to the top of the column considered.

cn = Component of the wind velocity normal to the shear vector

The scale has been constructed as follows:

The upper graph expresses the relationship between the variables (c, ϕ, and fc), $\frac{dc}{dz}$, ϕ and $f\frac{dc}{dz}$ or (Δc, ϕ and $f\Delta c$).

The horizontal lines are for values of c, Δc (labelled at the left in units of miles per hour) or dc/dz (labelled at the left in units of tenths of miles per hour per thousand feet). Velocity conversion scales have been included with Beaufort force at the left and units of meters per second and degrees of latitude per 24 hours at the right.

The vertical lines are for values of fc, $f\Delta c$ (labelled at the top in units of 10^{-2} cm/sec^2) or f dc/dz (labelled at the top in units of 10^{-3} cm/sec^2 1000 feet).

The sloping lines are for values of ϕ, labelled in the center. In order to cover the complete range of possible values the dashed sloping lines for ϕ have been entered for which the values of c, Δc, or dc/dz read from the graph should be multiplied by ten or the values of fc, $f\Delta c$, or f dc/dz read from the graph should be divided by ten.

The graph on the back expresses the relationship between the variables c, R, and c^2/R.

The horizontal lines are for values of c, labelled at the left in units of miles per hour, with conversion scales to units of meters per second and Beaufort force on the left.

The vertical lines are for values of c^2/R labelled at the bottom in units of 10^{-2} cm/sec^2.

The sloping lines are for values of R labelled at the top and the right in units of degrees of latitude. Conversion scales to units of miles and kilometers are included. In order to cover the complete range of values the dashed sloping lines for R have been entered, for which the values of a c read from the graph should be multiplied by ten or the values of c^2/R read from the graph should be divided by 100.

The lower graph expresses the relationship between T, p, and ρ.

The horizontal lines are for values of T labelled in Centigrade units at the left. A conversion scale to Fahrenheit units is at the right.

The sloping lines are for values of p, labelled at the bottom in units of millibars.

The vertical lines are for values of ρ, labelled at the bottom in units of 10^{-4} gm/cm^3.

The sloping line labelled "Index for T" reflects the vertical temperature scale into a horizontal temperature scale.

The top scale of the slide is for values of Δ n in units of degrees of latitude and is also for values of c_n in units of 10 miles per hour. The next two scales on the slide are conversion scales of Δ n to units of miles and kilometers. The next to bottom scale of the slide is for values of Δ p in units of millibars and is also for values of Δ T in Centigrade units. The bottom scale of the slide is for values of $\frac{\partial \Delta p}{\partial t}$ in units of millibars per three hours and is also for values of $\frac{\partial T}{\partial t}$ in units of degrees Centigrade per three hours.

The upper and lower graphs and the scales on the slide are so placed with respect to each other that:

When the value of Δ p is placed opposite the value of ρ, the value of Δ n will be opposite the value of fc, c^2/R or fc $\pm$ c^2/R on the fc scale.

When the value of Δ T is placed opposite the intersection (on the ρ scale) of the line for the value of T and the index for T, the value of Δ n will be opposite the value of f dc/dz.

When the value of c_n is placed opposite the value of f dc/dz, the value of $\frac{\partial T}{\partial t}$ will be opposite the intersection (on the ρ scale) of the line for the value fo T and the index for T.

When the value of c_n is placed opposite the value of f Δ c, the value of $\frac{\partial \Delta p}{\partial t}$ will be opposite the value of ρ.

All the scales are plotted for the logarithms of the values so the labelled values may be multiplied by any power of ten. Thus the entire range of all possible values of all the variables except T is covered. The values, as labelled, provide a convenient means of determining the correct placement of the decimal points.

It will be noted that usually it will not be necessary to take the time to read the values of ρ , fc, f Δ c, or f dc/dz in using the scale, as the positions of the lines for the values of these quantities is all that is required to set the scale or to read the value of the desired answer.

Following are some examples of and suggestions for the use of the scale:

PRESSURE GRADIENT RELATIONSHIPS

For much work the percentage variation of the density of the air at a given level will be small enough that a mean value of the density may be as used at that level. For example, for rough work on all sea-level maps drawn with Δ p = 3 mb, the slide may be set once and for all with Δ p = 3 mb opposite the intersection of the T = 10°C line and the p = 1013 mb line (at ρ = 12.6 x 10^{-4} gm/cm^3). Also, for rough work on all 10,000 maps, the slide may be set for good with the value of Δ p used opposite the intersection of the T = 0°C and p = 700-mb lines. (At ρ = 8.9 x 10^{-4} gm/cm^3).

Greater accuracy with little if any loss of time can be obtained by setting the slide once for each map worked on. A glance at the map should suffice to determine approximate mean values of the temperature and pressure over the map to use for this setting. Even greater accuracy could be obtained with little more time involved by setting the slide for regions of a given map. For very high level maps it will probably be desirable to set the slide for each point considered since for these levels percentage variations in density are quite large:

1. Geostrophic Wind Equation.

With the slide set with the value of Δ p opposite ρ, the scale is in effect a graph of c plotted as a function of Δ n for various values of ϕ. Examples: Slide set for a sea level map with $\Delta p = 3$ mb, p = 1013 mb, $T = 10^\circ$C.

Given	Answer
c = 30 mi/hr, $\phi = 30^\circ$	$\Delta n = 2.2^\circ$ lat. = 151 mi = 245 km.
c = 100 mi/hr, $\phi = 20^\circ$	$\Delta n = 0.97^\circ$ lat. = 66 mi = 107 km.
c = 6 mi/hr, $\phi = 38^\circ$	$\Delta n = 9.0^\circ$ lat. = 630 mi = 1010 km.
Δn = 3° lat, $\phi = 50^\circ$	c = 14.3 mi/hr = Beaufort 4 = 6.4 m/s = 5.0° lat/day
Δn = 600 mi, $\phi = 49^\circ$	c = 5 mi/hr = 2.2 m/s = 1.7° lat/day

2. Cyclostrophic Wind Equation.

The value of c^2/R can be read from the graph on the back, and opposite this value on the fc scale the spacing, Δ n, can be read, and vice versa.

Example: Slide set for $T = 40^\circ$C, p = 950 mb., $\Delta p = 3$ mb.

Given: $R = 1^\circ$ lat., c = 100 mi/hr.

Then $c^2R = 180 \times 10^{-2}$ cm/sec^2 and $\Delta n = 0.142^\circ$ lat. = 9.8 mi = 15.8 km.

Given: n = 12 mi, R = 100 mi.

Then $c^2/R = 147 \times 10^{-2}$ cm/sec^2 and c = 110 mi/hr. = 49 m/s.

3. Gradient Wind Equation.

If c is known, fc can be read on the upper graph and c^2/R can be read on the graph on the back. Then, with the slide set for the correct values of T, p, and Δ p, the value of Δ n can be read opposite the value of $fc \pm c^2/R$ on the fc scale.

Example: Slide set for $p = 4$ mb, $p = 700$ mb, $T = 0^{\circ}C$.

Given: $c = 50$ mi/hr, $\phi = 50^{\circ}$, $R = 5^{\circ}$ Lat.

Then $fc = 25 \times 10^{-2}$ cm/sec^2.

$c^2/R = 9$ "

$fc \pm c^2/R = 34$ " for cyclonic curvature.

$= 16$ " for anticyclonic curvature.

Then $\Delta n = 1.19^{\circ}$ lat. if the curvature were cyclonic.

or $\Delta n = 2.51^{\circ}$ lat. if the curvature were anticyclonic.

If the spacing of the isobars, Δn is known, the value of $fc \pm \frac{c^2}{R}$ can be read opposite Δn and the value of c determined by successive approximations, using the upper graph and the graph on the back.

Example: Slide set for $\Delta p = 4$ mb, $p = 700$ mb, $T = 0^{\circ}C$

Given: $\Delta n = 2.0^{\circ}$ lat., $\phi = 50^{\circ}$ $R = 70^{\circ}$ lat. and is cyclonic.

Then $fc + c^2/R = 20.1 \times 10^{-2}$ cm/sec^2.

We may take the geostrophic wind as the first approximation. Then

$fc_1 = 20.1$ giving $c_1 = 41$ mi/hr for which $c_1^2/R = 4.3$

Then $fc_2 = 20.1 - 4.3 = 15.8$: $c_2 = 32$ mi/hr: $c_2^2/R = 2.6$

$fc_3 = 20.1 - 2.6 = 17.5$: $c_3 = 35$ mi/hr: $c_3^2/R = 3.2$

$fc_4 = 20.1 - 3.2 = 16.9$: $c_4 = 34$ mi/hr: $c_4^2/R = 3.0$

$fc_5 = 20.1 - 3.0 = 17.1$: $c_5 = 34.5$ mi/hr: Answer

THERMAL GRADIENT RELATIONSHIPS

The values of dc/dz, Δc, and c_n, are very conveniently found from hodographs of pilot-balloon observations since the hodograph also shows clearly the direction of the isotherms, the sense of the thermal gradient and the sign of the temperature and pressure advections. It should be noted that the height of the column for which Δc is measured does not enter into the calculations, but that dc/dz is used in units of miles per hour per thousand feet. Since a mean value of the shear from sea level to 10,000 feet is frequently desired, it is helpful to note that the dc/dz scale is labelled in units of tenths of miles per hour per thousand feet which is equivalent to miles per hour per ten thousand feet.

In the absence of simultaneous radiosonde data to determine the values of T, any reasonable assumption of the values of T will introduce but a small percentage error, since the possible ranges of T give small percentage variations on the absolute scale of temperature. Monthly mean values for the station considered would give quite accurate results.

4. Thermal Gradient Equation.

The slide is set with the desired value of ΔT placed opposite the intersection (on the C scale) of the line for the value of T and the Index for T. With the slide set the scale is in effect a graph of dc/dz plotted as a function of f dc/dz for various values of ϕ.

Example: Slide set for $\Delta T = 4^\circ C$, $T = 0^\circ C$.

Given	Answer
dc/dz = 4 mi/hr/1000 ft, $\phi = 42^\circ$	$\Delta n = 2.27^\circ$ lat.
dc/dz = 20 mi/hr/1000-ft, $\phi = 20^\circ$	$\Delta n = 0.88^\circ$ lat.
$\Delta n = 3^\circ$ lat, $\phi = 50^\circ$	dc/dz = 2.62 mi/hr/1000 ft.

5. Advective Temperature Change.

(NOTE: On the first printing of the scale the formula, the instructions and the label $\frac{\partial T}{\partial t}$ —— °C/3 hr (on the $\frac{\partial \Delta p}{\partial t}$ —— mb/3 hr scale) for this use were omitted)

The slide is set by placing the value of c_n opposite the line of the value of f dc/dz as determined by the intersection of the lines for the values dc/dz and ϕ. The value of $\frac{\partial T}{\partial t}$ can then be read opposite the intersection (on the C scale) of the line for the value of T and the Index for T.

Example: Given: dc/dz = 3.5 mi/hr/1000 ft, c_n = 25 mi/hr. $\phi = 42^\circ$, $T = 0^\circ C$

Then $\frac{\partial T}{\partial t} = 1.66^\circ C/3$ hr.

6. Advective Pressure Change

The slide is set by placing the value of c_n opposite the line for the value of $f \Delta c$ as determined by the intersection of the lines for the values of Δc and ϕ.

Then the value of $\frac{\partial \Delta p}{\partial \Delta t}$ can be read opposite the value of ρ as determined by the intersection of the lines for the value of T and p. In the absence of any more specific values of T and p, a value of ρ - 10.9×10^{-4} (for $T = 0^{\circ}C$, $p = 850$ mb) may be used for the mean value of the density of the whole column in the common problem of finding $\frac{\partial \Delta p}{\partial \Delta t}$ for the column from sea-level to 10,000 feet.

Example: Given $\Delta c = 30$ mi/hr, $c_n = 16$ mi/hr, $\phi = 42^{\circ}$

Assume $T = 0^{\circ}C$, $p = 850$ mb,

Then $\frac{\partial \Delta p}{\partial \Delta t} = 1.09$ mb/3 hr.